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Developing Mathematical Vocabulary

Specialized Mathematical Vocabulary for All Learners

In mathematics, vocabulary is highly specialized. These words are often not encountered in everyday life. Therefore, all students need an explicit introduction and explanation of these vocabulary words in order to be able to apply them to their understanding of mathematical concepts. The task is even more difficult for English language learners. These vocabulary words are not typically the words that English language learners will learn during their structured English Language Development class period. Therefore, it is up to the content teacher

who teaches mathematics to make certain that English language learners learn the necessary vocabulary in order to achieve comprehension of mathematical concepts and curriculum.

Furthermore, the different areas of mathematics (e.g., number sense and mathematical reasoning) and the various disciplines (e.g., algebra, trigonometry, geometry, and calculus) have different compilations of specialized vocabulary words. Sometimes there is overlap with words across mathematical areas and disciplines, but often there are new words that are specific to just one mathematical area or discipline. It is vital to understand the vocabulary for a specific discipline in mathematics because this knowledge aids in access to the core curriculum. All students are required to demonstrate mastery of the concepts, and this will only be possible if they first achieve understanding of the vocabulary words that explain, describe, and facilitate each of the mathematical concepts.

Vocabulary Development for English Language Learners

It is important for all students learning mathematics to be familiar with the specialized vocabulary embedded within the practice and application of the concepts. English language learners especially need consistent, structured instruction in learning English (English Language Development). They also need well-planned, sheltered instruction throughout content lessons and effective activities to develop mathematics vocabulary (Dean and Florian, 2001).

It is not enough to give the students a list of words and have them look up the definitions in dictionaries or textbook glossaries. Students who are struggling with learning a language are not going to find the process easier by simply being given more words to sort through. What

English language learners need are context-embedded lesson activities that acquaint them with the necessary words for comprehension of the content and allow them to practice the use of the words in activities that span listening, speaking, reading, and writing actions.

Mathematics teachers need to be cognizant of the language difficulties students have who are learning English. Many mathematics teachers believe English instruction is the job of the English teacher. However, the English teacher is not focusing on the specialized mathematics vocabulary and the contexts appropriate to it during English class. There are other necessary language components for the students to learn at that time. Therefore, it is necessary for the mathematics teacher to offer the scaffolding students need for access to mathematical concepts. By knowing the language level of each individual student, the teacher can plan appropriate lessons that balance vocabulary development, instruction, modeling, interactive activities, and support.

Types of Language Proficiency

One major concept mathematics teachers need to recognize is the difference between the two types of language proficiency for English language learners. Jim Cummins coined the two types of language Basic Interpersonal Communication Skills (commonly referred to as BICS) and Cognitive Academic Language Proficiency (commonly referred to as CALP) (Crawford, 2004). BICS refers to a student's *social language*. Proficiency in social language requires no specific instruction and typically takes as little as three years to acquire. This knowledge can be acquired through media saturation, music, and social situations. Students can easily seem very capable in social language because they need it to survive. A teacher can often be tricked by a student's level of BICS. The teacher may hear a student chatting with friends and converse with that student before or after class. These conversa-

tions may lead the teacher to believe that the student has a firm grasp of the English language. However, that same student is failing assessments, struggling to keep up with assignments, and unable to write well about mathematical content. This student lacks CALP.

CALP, or *academic language*, takes seven or more years to acquire. CALP is proficiency in the language of the content areas and of the classroom. A student who has strong CALP has command of the use of English within content areas. In mathematics, a student with a strong level of CALP is able to understand key vocabulary, use it in correct context, and write well about his or her understanding of mathematical concepts and procedures. This level of academic language is not learned easily and intuitively, like BICS. This language proficiency only comes with explicit instruction and planned objectives by the content teachers. That is one reason why vocabulary-development lessons are so important for teachers of English language learners to incorporate into mathematics lessons.

Levels of Language Acquisition

Effective mathematics teachers of English language learners also need to know the levels of language acquisition for each English language learner in the classroom. The appropriate lesson for a student who has just moved into the country is going to look very different from the appropriate lesson for an English language learner who is close to being considered fluent in the English language.

Many states have official assessments meant to determine the level at which a student is able to use English. These assessments cover the areas of listening, speaking, reading, and writing. Some of the assessments have a separate score for each domain of language and then a composite score that combines the overall level at which the student is performing in English.

This chart gives information about the levels of language acquisition of English language learners and suggestions to teachers for how to meet those students' needs.

Beginning	Early Intermediate/ Intermediate	Early Advanced/ Advanced
These students fall into a wide range of limited English comprehension. They have minimal or limited comprehension with no verbal production. Some beginning students are just able to give one- or two-word responses. Some are beginning to comprehend highly contextualized information and are able to speak in very simple sentences.	These students have good comprehension of information in context. They may exhibit restricted ability to communicate ideas, but they can usually reproduce familiar phrases in simple sentences. As they improve in proficiency, they improve in the ability to communicate ideas, although they may exhibit errors in production, especially when writing or speaking about highly specialized content.	These students may "trick" teachers into thinking that they are fluent in English. But they often struggle when they have to explain their understanding of an answer or write out the procedures of a concept. They lack the ability to fully communicate higher levels of thinking in content-specific academic language.
Teachers should: • Provide a lot of context for mathematical concepts. • Use physical movement and visuals to explain mathematical vocabulary. • Use sentence frames to help students place mathematical concepts into context. • Ask yes/no questions or questions where the answers are embedded in the questions. • Always include vocabulary-development activities.	**Teachers should:** • Provide visuals and context for mathematical concepts. • Encourage cooperative and interactive activities in order to make mathematical content comprehensible. • Ask questions that require simple sentences with known vocabulary. • Elicit simple explanations and summaries. • Support writing and reading tasks. • Often include vocabulary-development activities and the proper ways to communicate, using the mathematics vocabulary.	**Teachers should:** • Provide structured group discussion of concepts before requiring individual practice and writing about mathematical reasoning. • Elicit explanations that analyze and synthesize mathematical information. • Model the higher levels of thinking with use of specialized vocabulary. • Regularly practice vocabulary-development activities and then take the students to the next levels of higher-level thinking using the vocabulary.

Integrating Vocabulary Development into Instruction

Interactive vocabulary-development activities should be regularly integrated in mathematics lessons in all classrooms. These types of activities are especially necessary for classrooms with English language learners, students struggling with mathematical concepts, or any students who have not shown mastery of the vocabulary.

Teachers should follow these guidelines before beginning to teach the vocabulary activities demonstrated in this resource.

- Decide how long to use one vocabulary activity before introducing a new one.
- Plan for extra teaching time when a new vocabulary activity is being introduced.
- Choose an appropriate activity in order to meet the allotted classroom time for the particular lesson.
- "Frontload" the lesson with vocabulary words before the students need to apply them during practice activities and problems.
- Revisit past vocabulary words in addition to current words, if a lesson requires them.
- Repeat the activity with the same words or new words if it needs to be practiced a few times before the students can correctly perform the activity.
- Clearly state the purpose for an activity, the behavior expectations, and the consequences for not following the expectations if students will be out of their seats.

Vocabulary-Development Activities

These activities often cover the four domains of language—listening, speaking, reading, and writing. It is

Teaching Problem Solving

Why Teach Problem Solving?

Today's world is changing rapidly. Many of the changes mean that proficiency in basic mathematical concepts will become more and more critical.

Furthermore, mathematical reasoning and problem solving will be crucial to the success of today's students as they work to find solutions to problems in everyday life. Just knowing the basic facts and formulas is not enough to solve the wide range of problems and situations that arise in life. It is no surprise, then, that problem solving is an important, current mathematics focus in the classroom.

While students must be skilled at performing computations required to find mathematical solutions, this is only part of the process. Before students begin to manipulate the information in a problem, they should understand its meaning and plan a way to solve it.

Students, therefore, need to learn about problem solving as a process and the strategies they can apply to find solutions (Kilpatrick, et al., 2001). The process of problem solving goes beyond finding simple solutions; it encourages students to reflect on the solutions, make generalizations, and extend problems to include new possibilities for investigation. Once students learn the process of problem solving, they can use mathematical approaches to solve real-life problems.

The pages that follow provide explanations and examples of 12 problem-solving strategies that can be adapted to meet students' needs. Information about each strategy will provide insight into ways a particular strategy can be used in the classroom. Examples are given for each strategy. They demonstrate the application of the strategy to the solution of the problem. These examples are not appropriate across all grade levels. They are only used to demonstrate the use of the strategy.

Steps for the Problem-Solving Process

It is important that students follow a logical and systematic approach to their problem solving. These four steps will enable students to tackle problems in a structured and meaningful way. These steps are not intuitive for learners. Therefore, teachers will need to plan instructional time to explicitly teach the process, model it, and finally allow for ample opportunities for guided and individual practice as students approach the problem-solving strategies.

Step One

Understanding the Problem

Encourage students to read the problem carefully a number of times until they fully understand what it asks. As students are learning this step and progressing toward internalizing it, the teacher will allow time for students to discuss the problem with peers or rewrite the problem in their own words. Students should ask internal questions such as, "What is the problem asking me to do?" and "What information is relevant and necessary for solving the problem?" (This will need to be repeatedly modeled for the students in the learning process.)

Next, students should underline any unfamiliar words and find their meanings. Selecting the information they know and deciding what is unknown will help them begin too see how to solve the problem. They should also see if there is any unnecessary information. It will be helpful for teachers to model these processes until students understand how to complete them on their own.

Step Two

Planning and Communicating a Solution

Students should decide how they would solve the problem by thinking about the different strategies that could be used. Sometimes it will be necessary for students to use more than one strategy to solve a problem

They could try to make predictions, or guesses, about the problem. Often these guesses result in generalizations, which help to solve problems. Students should be discouraged from making wild guesses, but they should be encouraged to take risks. They should always think in terms of how this problem relates to other problems that they have solved.

As they attempt different strategies, they should keep a record of those they have tried so that they do not repeat them.

The 12 strategies in this book include:

- drawing a diagram
- drawing a table
- acting it out or using concrete materials
- guessing and checking
- creating an organized list
- looking for a pattern
- creating a tree diagram
- working backwards
- using simpler numbers
- open-ended problem solving
- analyzing and investigating
- using logical reasoning

Other strategies include:

- breaking down ideas into smaller pieces
- writing a number sentence
- writing down ideas as work progresses so students do not forget how the problem was approached
- approaching the problem systematically
- rereading the problem in order to rethink strategies if the student becomes "stuck"
- orally demonstrating and explaining how an answer was reached

Step Three

Reflecting and Generalizing

Many times the solution and strategies in one problem can help students know how to solve another problem.

Therefore students need to learn the importance of reflecting on the work they have done. Teachers need to teach students the critical process of reflection. This process should be modeled as teachers show problem solutions. Teachers can even solve problems incorrectly in order to go through the reflection process and "catch" mistakes. Students need to decide if their answers make sense and if they have answered the question that was asked. They should illustrate and write their thinking processes, estimations, and approaches. This gives them time to reflect on their practices and grow in the use of problem-solving strategies. When they have an answer, they should explain the process to someone else.

Step Four

Extension

This step also needs to be explicitly taught and modeled by teachers because students need practice in internalizing it. Students need to learn how to ask themselves "what if" questions to link this problem to another. This will take their explorations to deeper levels and encourage their use of logical thought processes. Students should also decide if it is possible to solve the problem in a simpler way.

Problem-Solving Strategies

Strategy One—Drawing a Diagram

This strategy often reveals aspects of the problem that may not be apparent at first. A diagram that uses simple symbols or pictures may enable students to see the situation more easily and can help them keep track of the stages of a problem in which there are many steps. Students need to develop the skills and understanding to use diagrams effectively.